11.3 Day 1`

Wednesday, May 22, 2019 8:52 AM

Precalculus Honors
11.3 Day 1

Name________________________

What is a limit?

Let's look at the graph of a few functions and look at some limits.

a. $\lim\limits_{x\to 3} 2x-1 = 5$

$2(3)-1=5$

b. $\lim\limits_{x\to\infty} \frac{1}{x} = 0$

c. $\lim\limits_{x\to 2} \frac{x^2+x-6}{x-2} = 5$

$\lim\limits_{x\to 2} \frac{(x+3)(x-2)}{(x-2)}$

$\lim\limits_{x\to 2} x+3$

$2+3=5$

Let's revisit a and c. Determine if the functions are continuous at the point in question.

Definition of a limit:

DEFINITION (INFORMAL) Limit at *a*

When we write "$\lim\limits_{x\to a} f(x) = L$," we mean that $f(x)$ gets arbitrarily close to L as x gets arbitrarily close (but not equal) to a.

Properties of limits:

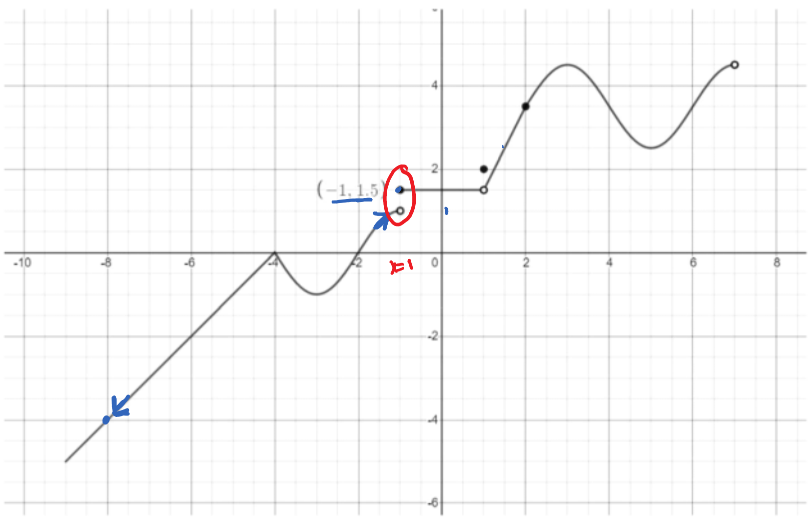
Sum Rule	$\lim\limits_{x\to c}(f(x)+g(x)) = \lim\limits_{x\to c} f(x) + \lim\limits_{x\to c} g(x)$
Difference Rule	$\lim\limits_{x\to c}(f(x)-g(x)) = \lim\limits_{x\to c} f(x) - \lim\limits_{x\to c} g(x)$
Product Rule	$\lim\limits_{x\to c}(f(x)\bullet g(x)) = \lim\limits_{x\to c} f(x) \bullet \lim\limits_{x\to c} g(x)$
Constant Multiple Rule	$\lim\limits_{x\to c}(k\bullet f(x)) = k\bullet\lim\limits_{x\to c} f(x)$
Quotient Rule	$\lim\limits_{x\to c}\frac{f(x)}{g(x)} = \frac{\lim\limits_{x\to c} f(x)}{\lim\limits_{x\to c} g(x)}$
Power Rule	$\lim\limits_{x\to c}(f(x))^n = (\lim\limits_{x\to c} f(x))^n$, for n is a positive integer
Root Rule	$\lim\limits_{x\to c}\sqrt[n]{f(x)} = \sqrt[n]{\lim\limits_{x\to c} f(x)}$ *for* $n\ge 2$ *and* $\lim\limits_{x\to c}\sqrt[n]{f(x)}$ are real numbers

THEOREM One-Sided and Two-Sided Limits

A function $f(x)$ has a limit as x approaches c if and only if the left-hand and right-hand limits at c exist and are equal. That is,

$$\lim_{x \to c} f(x) = L \Leftrightarrow \lim_{x \to c^-} f(x) = L \text{ and } \lim_{x \to c^+} f(x) = L.$$

Examples:



Above is the graph of $f\ x$

a. $\lim_{x \to -8^+} f(x) = -4$

b. $\lim_{x \to -1^-} f(x) = 1$

$\lim_{x \to -1} f(x) = DNE$

c. $\lim_{x \to 1} f(x) = 1.5$

$f(1) = 2$

d. $f(1.5) = 2.5$

e. Is $f(x)$ continuous at x=1. NO

Definition of continuity at a point (a) on a function:

1. $\lim_{x \to a^-} f(x) = L$ $\lim_{x \to a^+} f(x) = L$ $\lim_{x \to a} f(x) = L$ 2. $f(a) = L$ 3. $\lim_{x \to a} f(x) = f(a)$

Why or why not (using the definition of continuity) is $f(x)$ continuous at:

a. x= -1

No

B/C $\lim_{x \to -1} f(x) = dne$

b. x= -4

$\lim_{x \to -4^-} f(x) = 0$ $\lim_{x \to -4^+} f(x) = 0$ $f(-4) = 0$

yes $\lim_{x \to -4} f(x) = f(-4) = 0$

Example: Assume $\lim_{x \to a} f(x) = -3$ and $\lim_{x \to a} g(x) = 5$. Find the limit.

a. $\lim_{x \to a} (2f(x) + 3)$

$2 \lim_{x \to a} f(x) + \lim_{x \to a} 3$

$2(-3) + 3 \Rightarrow -3$

b. $\lim_{x \to a} \dfrac{g(x)}{1 - f(x)}$

$\dfrac{\lim_{x \to a} g(x)}{\lim_{x \to a} 1 - \lim_{x \to a} f(x)} \Rightarrow \dfrac{5}{1-(-3)} \Rightarrow 5/4$

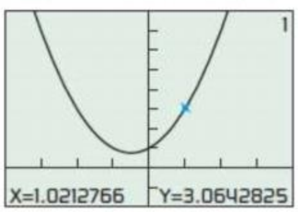
Numerical, Graphical, and Algebraic approaches to limits: (using the table feature on your calculator)

EXAMPLE 1 Finding a Limit

Find $\lim_{x \to 1} \dfrac{x^3 - 1}{x - 1}$.

Solve Graphically

The graph in Figure 10.11a suggests that the limit exists and is about 3.



[-4, 4] by [-2, 8]

(a)

FIGURE 10.11a A graph of $f(x) = (x^3 - 1)/(x - 1)$.

Solve Numerically

The table also gives compelling evidence that the limit is 3.

X	Y1	
.997	2.991	
.998	2.994	
.999	2.997	
1	ERROR	
1.001	3.003	
1.002	3.006	
1.003	3.009	

Y1 = (X³–1)/(X–1)

(b)

FIGURE 10.11b A table of values for $f(x) = (x^3 - 1)/(x - 1)$.

Solve Algebraically

$$\lim_{x \to 1} \frac{x^3 - 1}{x - 1}$$

$$= \lim_{x \to 1} \frac{(x - 1)(x^2 + x + 1)}{x - 1}$$

$$= \lim_{x \to 1} (x^2 + x + 1)$$

$$= 1 + 1 + 1$$

$$= 3$$

Use your calculator to determine the following limits:

a. $\lim_{x \to 0} \dfrac{\sin x}{x} = 1$

b. $\lim_{x \to 0} \dfrac{\sin (5x)}{(5x)} = 1$

c. $\lim_{x \to 0} \dfrac{\sin 5x}{x} = 5$

$\lim_{x \to 0} \dfrac{5}{5} \cdot \dfrac{\sin(5x)}{x} = \lim_{x \to 0} 5 \cdot \dfrac{\sin 5x}{5x}$

$5 \cdot 1 = 5$

LIMIT TO MEMORIZE: $\lim_{x\to 0}\frac{\sin\ ax}{ax}=1$

Steps for finding limits algebraically:

1. substitute
2. Algebraic work to rewrite the statement
3. substitute

Examples:

a. $\lim_{x\to 3}\sqrt{2x-1}=$ $\sqrt{2(3)-1}$ $=\sqrt{5}$

b. $\lim_{x\to -3} x^3-1=$ $(-3)^3-1=-28$

c. $\lim_{x\to 0}\frac{\sin\ 2x}{x}=\frac{2}{2}$

$\lim_{x\to 0} 2\cdot\frac{\sin 2x}{2x} \Rightarrow \lim_{x\to 0} 2\cdot\underbrace{\lim_{x\to 0}\frac{\sin 2x}{2x}}$

$2\cdot 1$

$2\cdot 1=2$

d. $\lim_{x\to 0}\frac{\sin 3x}{5x}$ $=\lim_{x\to 0}\frac{1}{5}\cdot\frac{3}{3}\frac{\sin 3x}{x}=3/5$

Limits as the graph of a function approaches asymptotes.

a. $\lim_{x\to 1}\frac{(x^2-1)}{(x^2-6x)}=$

b. $\lim_{x\to 0^+}\frac{(x-1)^2}{x}$

PIECEWISE FUNCTIONS AND LIMITS

Examples

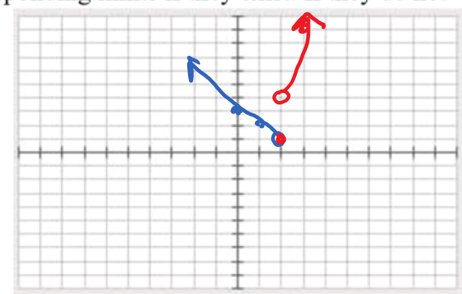
Graph the following piecewise functions and then find the corresponding limits if they exist. If they do not exist, explain why.

$$f(x)=\begin{cases}3-x & x<2\\ 1 & x=2\\ x^2 & x>2\end{cases}$$

$\lim_{x\to 2^+} f(x)=$ 4

$\lim_{x\to 2^-} f(x)=$ 1

$\lim_{x\to 2} f(x)=$ dne



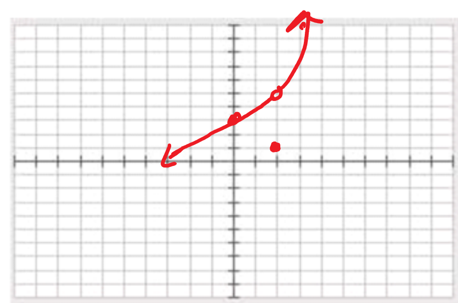
Is $f(x)$ continuous at $x=2$? not cont. b/c $\lim_{x\to 2} f(x) =$ dne

$$f(x)=\begin{cases}x+3 & x<2\\ 1 & x=2\\ x^2+1 & x>2\end{cases}$$

$\lim_{x\to 2^+} f(x)=$ 5

$\lim_{x\to 2^-} f(x)=$ 5

$\lim_{x\to 2} f(x)=$ 5



Is $f(x)$ continuous at x=2? NO

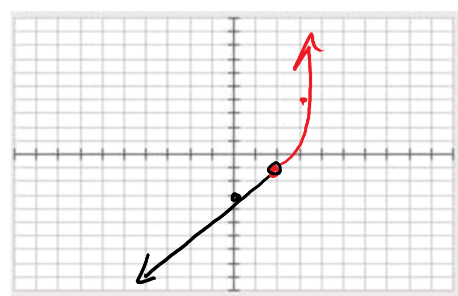
b/c $\lim_{x\to 2} f(x) \neq f(2)$

$$f(x)=\begin{cases}x-3 & x<2\\ -1 & x=2\\ x^2-5 & x>2\end{cases}$$

$\lim_{x\to 2^+} f(x)=$

$\lim_{x\to 2^-} f(x)=$

$\lim_{x\to 2} f(x)=$



Is $f(x)$ continuous at x=2?

yes $\lim_{x\to 2} f(x) = -1$ $f(2)=-1$

since $\lim_{x\to 2} f(x) = f(2)$

f(x) is continuous @ x=2